# On the relative stabilities of dodecahedron-shaped and bowl-shaped structures of $C_{20}$

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# Abstract

Purely carbonaceous aggregates,  $C_{20}$ , were studied by the AM1 quantum-chemical method. In addition to one dodecahedron-shaped structure possessing  $C_1$  symmetry, another three-dimensional species was revealed, namely a bowl-shaped structure of  $C_{5v}$  symmetry. The temperature dependence of the relative stabilities of both structures is evaluated, showing that the fullerenic species prevails in the temperature region studied. However, at very high temperatures, a relative-stability interchange is predicted. In view of the absence of any symmetry in the dodecahedron-shaped structure, the species is treated both as an equimolar mixture of two enantiomers and as one pure optical form. The effects of the enantiomerism on the overall properties of  $C_{20}$  are briefly discussed.

#### INTRODUCTION

A new class of molecules, fullerenes, has recently been established [1,2] as being a purely carbonaceous species  $C_n$  with  $n \ge 20$ , possessing a polyhedral shape built from five- and six-membered rings. At first, research concentrated on the already recognized  $C_{60}$  and  $C_{70}$  aggregates [3-6]. Quite recently, other fullerenes have also been observed [7,8]. Computations based on quantum chemical methods have followed [9–13] the experimental developments and descriptions of some key structural, spectral and energetic features of some selected fullerenes have been presented. It has been pointed out [13–15] that fullerene isomerism can become a significant phenomenon during fullerene synthesis.

This report deals with the smallest member of the fullerene family,  $C_{20}$ , and compares its thermodynamic stability with another plausible structure built from carbon rings that are not, however, closed into a cage. The

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thermodynamic treatment is based on the results obtained using the AM1 quantum-chemical method [16].

# THE C<sub>20</sub> SPATIAL STRUCTURES

Topological reasoning [1,2] can prove that any fullerene always contains exactly  $f_5 = 12$  five-membered rings, and the number of six-membered rings  $f_6$  varies with the number of carbon atoms m

$$f_6 = m/2 - 10$$

(1)

Hence, the smallest fullerene,  $C_{20}$ , is built from just 12 pentagons (and no hexagons). Such a structure is known [17] from the hydrocarbon dodecahe-



Fig. 1. Schemes of the AM1 structures of the  $C_1$  dodecahedron-shaped (bottom) and  $C_{5v}$  bowl-shaped (top)  $C_{20}$ .

#### TABLE 1

Relative AM1 potential energy terms  $\Delta E$  for the  $C_1$  dodecahedron-shaped<sup>a</sup> and  $C_{5v}$  bowl-shaped structures<sup>b</sup> of  $C_{20}$ 

Species	$\Delta E$ (kJ mol <sup>-1</sup> )	
$\overline{C_1}$	0.0	
C <sub>5v</sub>	208.68	

<sup>a</sup> The energetics is the same for (+) or (-) species as well as for their equimolar mixture. <sup>b</sup> See Fig. 1.

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drane, to be derived from the polyhedron dodecahedron  $(I_h \text{ point group of symmetry})$ . However, in our calculations with the AM1 quantum-chemical method [16], the local energy minimum of the dodecahedron shape (Fig. 1) exhibited  $C_1$  symmetry, i.e. no symmetry element is present. This means that all the five-membered rings in the framework are somewhat distorted, apparently in order to compensate for the steric tension originating in the five-membered ring junctions.

There is another non-planar (though not fullerenic) structure which can be considered in connection with  $C_{20}$ : the structure related to the hydrocarbon corannulene [18]. The latter structure is composed of one five-membered and five six-membered rings arranged in a bowl-like shape. Indeed, AM1 geometry optimization revealed that there is a local energy minimum for  $C_{20}$ , possessing  $C_{5v}$  symmetry (Fig. 1). The bowl-shaped structure is higher in energy by about 210 kJ mol<sup>-1</sup> than the dodecahedron  $C_1$  form (Table 1).

Although there can be several minimum-energy structures of  $C_{20}$ , the two three-dimensional species still represent an interesting system. In view of the stability interchanges found in other carbon-aggregate systems [13–15], this limited  $C_{20}$  set should also be submitted to thermodynamic stability evaluation. In fact, the temperature of the relative-stability interchange between two isomers should be independent of the presence of other species in the reaction mixture, i.e. this result for the two-membered mixture can be transferred to more general multi-isomer equilibrium mixtures.

# THERMODYNAMIC STABILITY EVALUATION

In an *n*-membered equilibrium isomeric set at a non-zero temperature, all its members are present, in principle, although their relative populations can vary considerably with temperature. This sensitivity in the isomeric composition has recently been documented for diverse cluster systems [19-23]. Let us introduce mole fractions  $w_i$  of the components of the isomeric equilibrium mixture. In terms of the ground-state energies  $\Delta H_{0i}^{\oplus}$  (i.e. the standard enthalpies at 0 K) and the isomeric partition functions  $q_i$ , the mole fractions are given by [22,23]

$$w_{i} = \frac{q_{i} \exp\left[-\Delta H_{0,i}^{\ominus}/(RT)\right]}{\sum_{j=1}^{n} q_{j} \exp\left[-\Delta H_{0,j}^{\ominus}/(RT)\right]}$$
(2)

where R stands for the gas constant and T for the temperature. The partition functions  $q_i$  are determined from the AM1 calculations using the rigid-rotor and harmonic-oscillator approximation [22].

There is no symmetry element present in the  $C_1$  dodecahedron-shaped structure. Although there is no asymmetric carbon atom in the conventional sense (i.e. a carbon atom with four different substituents) present in the structure, the structure possesses chirality, i.e. it is not superimposable upon its mirror image. In this more general approach to chirality, it is not the asymmetric carbon atom but the dissymmetry of a structure as a whole that is important (dissymmetry, however, is most frequently caused by the asymmetric carbon atom). Dissymmetry indicates that there is no reflection symmetry, which is the case in only the  $C_n$  and  $D_n$  point groups of symmetry. In particular, the  $C_1$  dodecahedron species forms a pair of enantiomers ((+), (-) optical isomers). We can distinguish two situations: (i) an equilibrium mixture of one of the  $C_1$  enantiomers (either (+) or (-)) with the  $C_{5v}$  isomer; and (ii) an equilibrium mixture of both  $C_1$  optical isomers with the bowl-shaped species. Formally, we can introduce into eqn. (2) the chirality partition functions  $q_{\chi} = 1$  or  $q_{\chi} = 2$  for situations (i) or (ii), respectively.

One can distinguish two classes of thermodynamic terms for an isomeric mixture: overall, many-component thermodynamic terms  $\Delta X_T^{\oplus}$  and partial, one-component terms  $\Delta X_i^{\oplus}$ . It is convenient to study the isomerism effects by means of so-called isomerism contributions [22,23] to thermodynamic terms  $\delta X_1$ 

$$\delta X_1 = \Delta X_T^{\oplus} - \Delta X_1^{\oplus} \tag{3}$$

The most stable species (at the low-temperature limit) is chosen as the reference structure and labelled by i = 1. If we deal with a ground-state species composed of two enantiomers, there are two conventions that can be followed. Here, we shall use in the  $q_{\chi} = 2$  case, the equimolar enantiomeric mixture as the reference standard (i.e. we consider the enantiomeric effects but we do not treat the optical isomers as the individual components of the isomeric mixture). In this article we shall study the isomeric effects on heat capacity at constant pressure  $(X = C_p)$ ; for the respective formulae see ref. 23. The heat capacity case is particularly interesting because the full (relaxation) term  $\delta C_{p,1}$  can exhibit a distinct

temperature maximum, which can even be conserved in the overall standard heat capacity term  $C_p^{\oplus}$ .

# **RESULTS AND DISCUSSION**

Table 2 presents the temperature development of the equilibrium mole fractions  $w_i$  in the mixture of the  $C_1$  and  $C_{5v}$  isomers of  $C_{20}$ . Both situations ( $q_{\chi} = 1$  or 2) are considered. The  $C_1$  species prevails, even at the highest temperature considered. For low  $w_{C_{5v}}$  values, the  $C_{5v}$  populations in its equilibrium mixture with just one  $C_1$  enantiomer are about twice as high as in the three-membered mixture. This is a natural consequence of the form of eqn. (2).

From a purely thermodynamic point of view, we can investigate the relative stability proportions at still higher temperatures (laser heating can actually create even higher temperatures than those used in the first small carbon-cluster observations [24]). It turns out (see Table 3) that in such an extremely high temperature region, there is a relative-stability interchange. Beyond the temperature threshold, the  $C_{5v}$  species becomes more populated than one or both enantiomers. The crossing temperature is naturally lower for two-membered than for three-membered mixtures and the difference is quite large. In the high temperature region, we can observe temperature maxima in the isomerism contribution to heat capacity  $\delta C_{p,1}/C_p^{\oplus}$  of the overall standard  $C_p^{\oplus}$  term.

The results in Table 3 are interesting from a general thermodynamic point of view – optimizing a fullerene synthesis is still a considerably more complex kinetic problem. Moreover,  $C_{20}$  itself is apparently quite an unstable fullerenic system. However, there is another fullerenic system [13]  $(C_{50})$  for which the enantiomeric effects can be shown at considerably lower temperatures. The  $C_{50}$  system is composed of (at least) three minimum-energy structures of  $D_{5h}$ ,  $D_3$  and  $C_{2v}$  symmetries. The  $D_{5h}$  structure

# TABLE 2

Equilibrium mole fractions  $w_i$  (%) for the  $C_1$  dodecahedron-shaped <sup>a</sup> and  $C_{5v}$  bowl-shaped structures of  $C_{20}$  at selected temperatures

T (K)	$q_{\chi} = 1$		$q_{\chi} = 2$	
	$\overline{w_{C_1}}$	w <sub>C5v</sub>	$\overline{w_{C_1}}$	w <sub>C5v</sub>
1000	100.0	$6 \times 10^{-8}$	100.0	$3 \times 10^{-8}$
2000	99.98	0.02	99.99	$9 \times 10^{-3}$
3000	98.83	1.17	99.41	0.59
4000	91.20	8.80	95.40	4.60

<sup>a</sup> The  $C_1$  species is treated either as just one of the (+), (-) isomers ( $q_{\chi} = 1$ ) or as their equimolar mixture ( $q_{\chi} = 2$ ).

### TABLE 3

Specification of some distinguished points <sup>a</sup> in the temperature interplay of the  $C_1$  and  $C_{5v}$ isomers of C<sub>20</sub>

Type <sup>a</sup>	$q_{\chi}^{b}$	Т (К)	w <sub>C</sub> , ° (%)	<sup>w</sup> C <sub>5v</sub> ° (%)	$\frac{\delta C_{p,1}}{(\mathbf{J} \mathbf{K}^{-1})} \mathbf{M}$	$C_p^{\oplus}$ (J K <sup>-1</sup> mol <sup>-1</sup> )	$\frac{\delta C_{p,1} / C_p^{\Leftrightarrow}}{(\%)}$
Crossing	1	6366	50.0	50.0	32.4	512	6.3
	2	7720	50.0	50.0	22.0	503	4.4
$\delta C_{p,1}$ maximum	1	5203	70.7	29.3	40.2	519	7.8
	2	5883	73.4	26.6	29.6	509	5.8
$\delta C_{p,1}/C_p^{\Theta}$ maximum	1	5197	70.9	29.1	40.2	519	7.8
	2	5876	73.5	26.5	29.6	509	5.8

That is, the point of two-isomer equimolarity (crossing), the maximum in the isomerism contribution to heat capacity  $\delta C_{p,1}$  and the maximum in the relative enhancement  $\delta C_{p,1}/C_p^{\Theta}$  of the  $C_p^{\Theta}$  term by isomeric interplay. <sup>b</sup> The  $C_1$  species is treated either as just one of the (+), (-) isomers (the upper line) or as

their equimolar mixture (the lower line).

<sup>c</sup> The mole fraction of the species in the equilibrium isomeric mixture.

Isomerism contribution to heat capacity related to the species more stable in the low d temperature region, i.e. to the  $C_1$  isomer(s).

#### TABLE 4

Enantiomeric effects on some distinguished points <sup>a</sup> in temperature interplay of the  $D_{\rm Sh}$ ,  $D_3$  and  $C_{2v}$  isomers of  $C_{50}$ 

Type <sup>a</sup>	$q_{\chi}^{b}$	Т (К)	w <sub>Dsh</sub> <sup>c</sup> (%)	w <sub>D3</sub> c (%)	w <sub>C2</sub> , ° (%)	$\frac{\delta C_{p,1}}{(\mathbf{J} \mathbf{K}^{-1})}^{\mathbf{d}}$ $\mathbf{mol}^{-1}$	$C_p^{\oplus}$ (J K <sup>-1</sup> mol <sup>-1</sup> )	$\frac{\delta C_{p,1} / C_p^{\Leftrightarrow}}{(\%)}$
Crossing	1	1390	50.0	50.0	$8 \times 10^{-57}$	24.9	1135	2.2
	2	1146	50.0	50.0	$1 \times 10^{-69}$	34.3	1095	3.1
$\delta C_{p,1}$ maximum	1	1110	69.3	30.7	$1 \times 10^{-71}$	29.9	1082	2.8
	2	970	66.7	33.3	$1 \times 10^{-82}$	38.9	1044	3.7
$\delta C_{p,1}/C_p^{\oplus}$ maximum	1	1070	72.5	27.5	$1 \times 10^{-74}$	29.7	1069	2.8
	2	932	70.5	29.5	$5 \times 10^{-86}$	38.6	1029	3.8

That is, the point of two-isomer equimolarity (crossing), the maximum in the isomerism contribution to heat capacity  $\delta C_{p,1}$  and the maximum in the relative enhancement  $\delta C_{p,1}/C_p^{\oplus}$  of the  $C_p^{\oplus}$  term by isomeric interplay. <sup>b</sup> The  $D_3$  species is treated either as just one of the (+), (-) isomers (the upper line) or as

their equimolar mixture (the lower line).

<sup>c</sup> The mole fraction of the species in the equilibrium isomeric mixture.

<sup>d</sup> Isomerism contribution to heat capacity related to the species most stable in the low temperature region, i.e. to the  $D_{5h}$  isomer.

has the lowest heat of formation, being followed after 30 and 1600 kJ mol<sup>-1</sup> by the  $D_3$  and  $C_{2\nu}$  isomers [13]. Incidentally, the second structure belongs to the dissymmetric  $D_3$  point group of symmetry which ensures that the structure cannot be superimposed on its mirror image. Thus, two enantiomers are again present. Table 4 compares the results [13] for  $q_{\chi} = 1$  with the  $q_{\chi} = 2$  case. It turns out that the inclusion of the other  $D_3$  enantiomer decreases considerably the three distinguished temperatures of the isomeric interplay. This is a consequence of the fact that the dissymmetric  $D_3$  species is not the ground-state structure in the  $C_{50}$  system.

It is to be expected that isomerism including optical isomerism will quite frequently be associated with fullerenic systems. A recent observation [25] has, in fact, provided clear experimental evidence that isomerism is present throughout carbon clusters. However, our finding optical isomerism in the  $C_1$  species is based on the semi-empirical AM1 quantum-chemical treatment, i.e. on a method of an approximate nature. Thus, it would certainly be useful to study the  $C_{20}$  fullerene structure at a non-empirical level as well, in order to avoid a methodical artifact.

#### REFERENCES

- 1 R.F. Curl and R.E. Smalley, Science, 242 (1988) 1017.
- 2 H. Kroto, Science, 242 (1988) 1139.
- 3 H.W. Kroto, J.R. Heath, S.C. O' Brien, R.F. Curl and R.E. Smalley, Nature, 318 (1985) 162.
- 4 W. Krätschmer, L.D. Lamb, K. Fostiropoulos and D.R. Huffman, Nature, 347 (1990) 354.
- 5 R. Taylor, J.P. Hare, A.K. Abdul-Sada and H.W. Kroto, J. Chem. Soc. Chem. Commun., (1990) 1423.
- 6 R.E. Haufler, J. Conceicao, L.P.F. Chibante, Y. Chai, N.E. Byrne, S. Flanagan, M.M. Haley, S.C. O'Brien, C. Pan, Z. Xiao, W.E. Billups, M.A. Ciufolini, R.H. Hauge, J.K. Margrave, L.J. Wilson, R.F. Curl and R.E. Smalley, J. Phys. Chem., 94 (1990) 8634.
- 7 F. Diederich, R. Ettl, Y. Rubin, R.L. Whetten, R. Beck, M. Alvarez, S. Anz, D. Sensharma, F. Wudl, K.C. Khemani and A. Koch, Science, 252 (1991) 548.
- 8 R. Ettl, I. Chao, F. Diederich and R.L. Whetten, Nature, 353 (1991) 149.
- 9 W. Weltner, Jr., and R.J. Van Zee, Chem. Rev., 89 (1989) 1713.
- 10 D. Bakowies and W. Thiel, J. Am. Chem. Soc., 113 (1991) 3704.
- 11 D. Bakowies and W. Thiel, Chem. Phys., 151 (1991) 309.
- 12 P.W. Fowler, J. Chem. Soc. Faraday Trans., 87 (1991) 1945.
- 13 Z. Slanina, L. Adamowicz, D. Bakowies and W. Thiel, Thermochim. Acta, 202 (1992) 249.
- 14 Z. Slanina, J.M. Rudziński, M. Togasi and E. Osawa, Thermochim. Acta, 140 (1989) 87.
- 15 Z. Slanina, J.M. Rudziński and E. Osawa, Z. Phys. D, 19 (1991) 431.
- 16 M.J.S. Dewar, E.G. Zoebisch, E.F. Healy and J.P. Stewart, J. Am. Chem. Soc., 107 (1985) 3902.
- 17 L.A. Paquette, Chem. Rev., 89 (1989) 1051.
- 18 W.E. Barth and R.G. Lawton, J. Am. Chem. Soc., 93 (1971) 1730.
- 19 M.R. Hoare, Adv. Chem. Phys., 40 (1979) 49.
- 20 Z. Slanina, Adv. Quantum Chem., 13 (1981) 89.

- 21 T.P. Martin, Phys. Rep., 95 (1983) 167.
- 22 Z. Slanina, Contemporary Theory of Chemical Isomerism, Academia and Reidel, Prague and Dordrecht, 1986.
- 23 Z. Slanina, Int. Rev. Phys. Chem., 6 (1987) 251.
- 24 J. Drowart, R.P. Burns, G. DeMaria and M.G. Inghram, J. Chem. Phys., 31 (1959) 1131.
- 25 G. v. Helden, M.-T. Hsu, P.R. Kemper and M.T. Bowers, J. Chem. Phys., 95 (1991) 3835.